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THE ELONGATION DUE TO MAGNETIZATION

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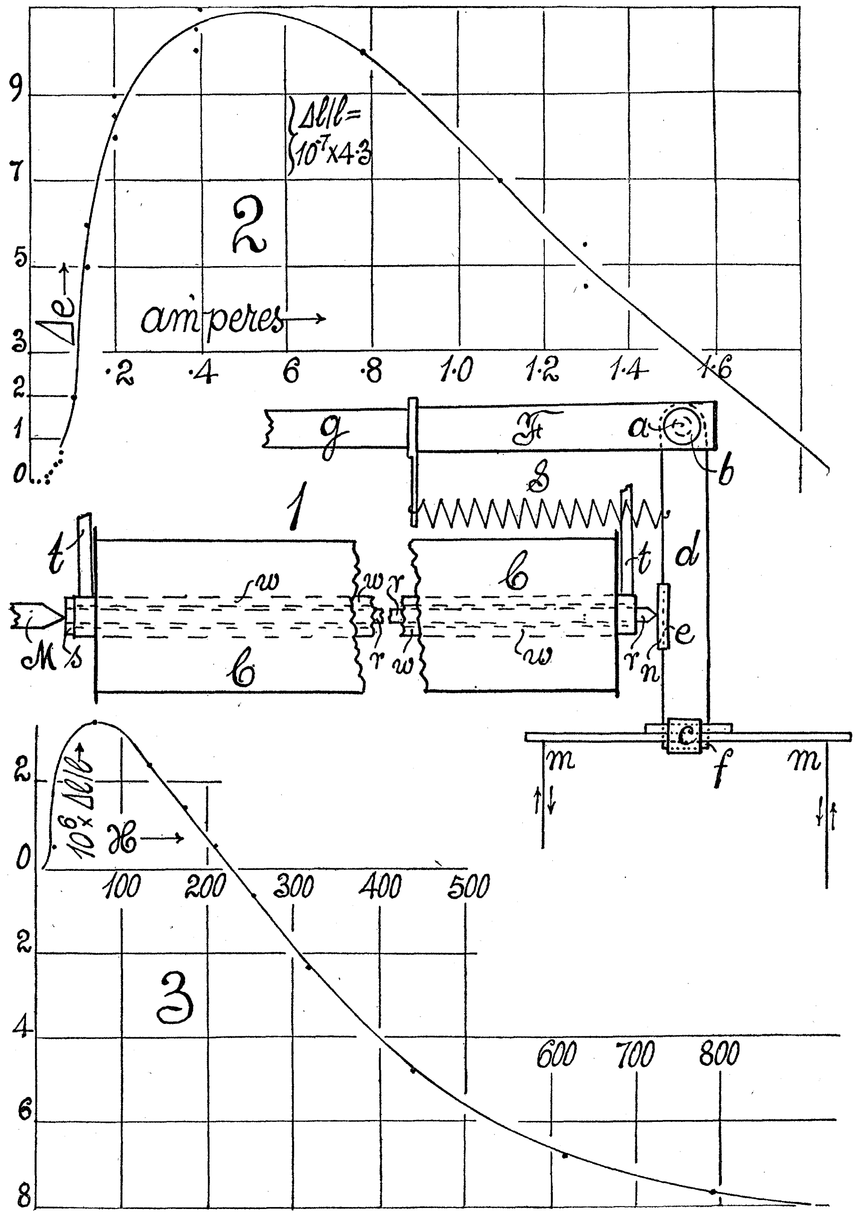
1. Introductory.—The small longitudinal displacements due to magnetostriction have been frequently subjected to investigation and an excellent summary is given in Winkelmann's Handbuch, vol. 5, p. 307, et seq., 1908. The measurements of Prof. C. G. Knott and his students Nagaoka and Honda are particularly noteworthy.[1] In 1911[2] my son, Mr. Maxwell Barus, and I used these phenomena for the purpose of testing a peculiar type of interferences then under discussion.

The present purpose is similar, being a test of the contact lever recently[3] described.

The elongation (and contraction) phenomena are necessarily complicated by the occurrence of hysteresis loops to which the present paper (in which the measurements are not made by the continuous variation of currents and field, but by successively making and breaking the circuit) will give no attention. This subject has been adequately explored by Professor Knott and the authors cited. The chief interest in this paper is rather the continued increase of the contractions due to magnetization, not only after the latter has practically reached a maximum, but in a marked degree (so far as I have gone, fields up to 800), indefinitely. There is no sure indication of an abatement of the contraction. Hence the magnetic contribution of the present paper is to lie in the treatment in strong fields.

2. Apparatus.—The contact lever shown in figures 1 and 2 of the paper cited was modified as indicated in figure 1, where F is the semicircular fork in a vertical plane, rigidly attached to the bed plate of the interferometer by a strong clutch (not shown), holding the cylinder, g, the handle of the fork. The vertical axis a of the contact lever is secured between the screw pivots b of the fork. The horizontal strip of brass d, rigidly fastened to the center of the axle a, carries at its end the auxiliary mirror mm' of the quadratic interferometer. For this purpose, a short length, f, at the end of d has been bent upward at right angles to d, so that mm' may be held between plates of brass by the yoke-shaped steel clip c. At the side of the lever is a vertical brass plate inset c, to which a small glass plate n has been fastened with cement. It is against this that the conical end of the iron rod rr to be examined, pushes. The spring S attached to the blade d and a lateral projection from the fork F insures continuous contact at a constant pressure.

The iron rod rr originally about 43 cm. long and 67 cm. in diameter, is enveloped by a tubular water jacket ww. Through this a current of water entering at t and leaving at t' is kept flowing from a large copper Mariotte flask about 50 cm. high and 30 cm. in diameter, and containing water at the tempera-


2
9
7
5
3
2
1
0
Δe →
amperes →
·2
·4
6
·8
1·0
1·2
1·4
1·6
Δl/l = 10⁻⁷ × 4·3
1
g
F
a
b
S
d
t
t
b
b
w
w
w
w
r
r
r
r
M
s
e
n
c
f
m
m
3
2
0
2
4
6
8
10⁶ × Δl/l
H →
100
200
300
400
500
600
700
800

ture of the room. Two such flasks were at hand to be used alternately. The coil *CC* 26 cm. long and 3.7 cm. in external diameter, is wound immediately on the tubular water jacket.

The rod *rr* fits the tube *ww* loosely and is centrally detached at the remote end by aid of a bushing *s* and a small bolt. The front end is free. The coil *CC* is held in position by a large clutch (not shown) encircling it at the middle and attached to the bed plate of the interferometer. It is additionally attached at the tubulures *t* and *t'*. Finally the conical end *M* of a micrometer screw (also rigidly attached to the bed plate) gives the remote end of the rod *rr* any desirable fiducial position. This micrometer *M* has the further advantage of permitting an independent standardization of the contact lever, as there is obviously always sufficient elastic yielding in the apparatus to considerably shift the interference fringes.

In the experiments made, the breadth of the ray rectangle *mm'* of the interferometer was $b = 9.7$ cm.; the normal distance between the rod *rr* and the axis *a*, 7 cm.; the length of the contact lever *a* to *mm'*, 10.6 cm.; and the length of the axle *a*, 10 cm.

3. *Observations.*—The helix *C* in figure 1 was slender in shape, the length being 37 cm. and the diameter within being about 1.5 cm. There were about 11.2 turns per centimeter per layer and 8 layers of wire so that the field within may be estimated at $H = 110\ i$ gauss, i being the current in amperes. The current 0.01 to over 8 amperes thus corresponded to field from 1 to over 800 gauss.

The first rod selected was of low carbon shop steel 43 cm. long, so that it projected a few centimeters beyond either end of the helix.

The displacement of fringes observed was characteristic, being (in the smaller fields) slow and deliberate on closing the circuit (so that their motion could almost be followed by the eye), but very rapid on breaking the circuit.

The experiments were begun with small fringes (about 0.1 mm. in the ocular), and the readings Δe were made in terms of an ocular micrometer scale which was a centimeter divided into 0.1 mm. This was compared with the datum, ΔN, of the displacement micrometer normal to one of the mirrors of the interferometer.

If ΔN corresponds to the angular displacement, $\Delta\theta$, of the contact lever and to Δl of elongation of the iron rod r in the helic C (figure 1) we may write as above

$$2b\Delta\theta = 2\Delta N \cos i \tag{1}$$

if i is the angle of incidence (45°) at the mirrors of the interferometer and b the breadth of the ray parallelogram. But

$$\Delta l = r\Delta\theta \tag{2}$$

if r is the normal distance of the line of thrust of the rod *rr* from the axis of the contact lever. Thus

$$\Delta l = (r \cos i/b)\ (\Delta N/\Delta e)\ \Delta e \tag{3}$$

If l is the length of the iron rod $\Delta l/l$ will be the datum required.

It was quite possible to ascertain $\Delta e = 0.1$; i.e., elongations of but $\Delta l/l = 5 \times 10^{-8}$, equivalent to $\Delta l = 2.3 \times 10^{-5}$ cm. The current must exceed 0.02 am. before any elongation can be detected, after which, however, the elongations abruptly begin and increase rapidly to a maximum, which is reached before saturation.

The experiments, figure 2, were made with somewhat greater care and with larger fringes. The standardization of the ocular micrometer showed

$$\Delta N/\Delta e = 10^{-5} \times 1.82, \quad \Delta l/l = 10^{-7} \times 2.16\, \Delta e$$

But for incidental difficulties (tremors, etc,), the results in figure 2 would probably be very smooth.

A number of supplementary experiments (see figure 2) were made to see whether the observed $\Delta l = 0$ for currents below 0.02 amperes might not be equivalent to an initial small minimum. But Δl remained persistently zero, while currents decreased from 0.02 to 0.001 amperes. At 0.004 am. the field was reversed, but no significant Δl could be observed. The fringes just moved ($\Delta e = 0.1$) when i was about 0.035 amperes, indicating a field of 3 or 4 gauss.

A rough test made of the equation by pushing the rod *rr* forward by the backstop screw *M*, figure 1, gave corroborative results.

If $\Delta\varphi$ refers to the turns of the screw *M*

$$\Delta l = (r \cos i/b)\ (\Delta N/\Delta e)\ (\Delta e/\Delta\varphi)\ \Delta\varphi$$

$(\Delta N/\Delta e)$ was found to be $10^{-5} \times 3.3$ cm. per scale part and $(\Delta e/\Delta\varphi)$ scale parts per degree of turn. Hence with the above data

$$\Delta l = 10^{-4} \times 1.8$$

The back screw having 40 turns to the inch, i.e., a pitch of 0.0635 cm. gives us $10^{-4} \times 1.76$ cm. per degree of turn.

Another feature may be mentioned here. The expansion of the coil when carrying very large currents is a thrust on the back stop *M*, which is quite appreciable and appears as an apparent contraction of the rod.

4. *Vibration telescope.*—To test the surmise that initial elongations always precede the final contraction, the vibration telescope heretofore described[4] was installed. It was then found that the even band of fringes drawn out by the vibrating objective broke up into strongly sinuous lines on making and particularly on breaking the circuit through the helix. When the circuit was made and broken alternately, the waves broke up further into a succession of discontinuous pulses of more than double the amplitudes of the waves. With the field properly adjusted by passing 1.8 to 2 amperes through the coil, there was no further observable displacement after the strong waves lines, produced immediately after closing the circuit, had subsided.

5. *Further observations.*—The data of figure 3 were investigated in a single series. To reduce the heat discrepancy a brisk current of water was passed through the tubular water jacket. This seemed the safer plan, even though the fringes were shaken. The observations were made in triplets and largely confined to the higher fields.

The curve is quite as clearly indicated as may be expected owing to the difficulties cited; but the higher observations ($H > 200$) are decreasing contractions. The reason of this is partly owing to the method of observation in triplets, where (curiously enough) the third reading (field zero) was a contraction in relation to the first reading in the absence of the field.

The apparatus in these experiments was therefore suspected of being faulty in design, inasmuch as the clutch of the contact lever and of the coil were attached to the same rigid standard. This arrangement was now modified, so that the two mountings were quite independent, whereupon the anomalous results specified largely receded.

As a second test a rod 28 cm. long of Swedish iron was inserted, the extra length being pieced out by brass tubing soldered to each end, so that the iron lay quite within the coil. The data obtained closely resembled figure 3. Tests made with other metals gave no positive results.

6. *Coefficient of expansion.*—To arrive at a definite reason for the occurrence of the anomalous contraction mentioned above, it seemed desirable to modify the magnetic apparatus for the purpose of measuring the coefficient of expansion of a given metal. This could easily be done by using the coil merely as a heater.

A number of experiments were made, using either the ocular micrometer (here the temperature increments must lie within 2°) and the Fraunhofer micrometer at the mirror of the interferometer. It was observed that all the expansions were apt to begin with a contraction immediately after the heating current had been closed. Hence there is an initial expansion of the coil itself. It was soon found that the consequent flexure of the table was the ultimate cause of the interferometer discrepancy.

A modification of the apparatus was made therefore by allowing the end C of the coil to recline on a large grooved wheel, which by slight rotation would admit of any expansion of the kind in question. With this improvement the anomalous contractions vanished and the work thereafter proceded smoothly.

7. *Theoretical observations.*—To account for such a graph as figure 3, as a whole, one may argue that the initial elongations are to be referred to the rotations of the molecular magnets. For these elongations are coextensive in field variations with the marked increase of magnetization. It would then seem plausible that thereafter the attractions between the oriented molecular magnets may be instanced to account for the persistent contractions in continually increasing fields. Thus it seems worth while to endeavor to ascertain whether such a supposition would conform with any reasonable value of the susceptibility k of the iron, which one may estimate as decreasing from over

100 to less than 10 as the rod approaches saturation ($H = 150$ gauss) and to decrease thereafter asymptotically to zero.

If p is the force per square centimeter of section of the rod and E Young's modulus,

$$p = E(\Delta l/l) \tag{1}$$

regarding the magnetic stress as traction.

Using the expression for the potential of a disc, the field F in a narrow crevasse normal to F, between molecular layers of magnetic surface density of magnetization kH

$$F = 4\pi kH + H$$

where k is the susceptibility of the metal.

Hence the force per square centimeter should be $p' = FkH$, or

$$p' = H^2(4\pi k^2 + k) \tag{2}$$

Equating $p = p'$ in equations (1) and (2)

$$\frac{\Delta l}{l} = \frac{4\pi k^2 + k}{E} H^2 \tag{3}$$

If the data in figure 3 are taken above 800 gauss, supposing that these are far enough removed from the initial complications, the estimate would be ($E = 2 \times 10^{12}$), $k = 1.6$.

An order of mean susceptibility of 1.6 (which seems not an unreasonable assumption) would thus account for the observed contractions. Naturally as k is essentially variable with H a better statement of the case might be given by postulating such a relation.

[1] *London, Phil. Mag.*, **37**, 1894, (131).

[2] *Carnegie Inst., Washington, Pub.*, No. 149.

[3] These PROCEEDINGS, **5**, 1919, (39).

[4] These PROCEEDINGS, **4**, 1918, (328).

GROUPS INVOLVING ONLY TWO OPERATORS WHICH ARE SQUARES

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The abelian group of order 2^m and of type (1, 1, 1, . . .) is completely characterized by the fact that all of it operators have a common square. When we impose the condition that the operators of a finite group G have two and only two distinct squares then G must belong to one of three infinite systems of groups whose characteristic properties we proceed to determine.